# Hyperbolic Geometry Homework 1

Name: ______________________________

1. (4 points) Let $\theta(t) = (t+1)\pi/3$ and $\gamma(t) = \cos(\theta(t)) + i\sin(\theta(t))$ for $0 \le t \le 1$. Compute the hyperbolic length
$$\int_\gamma \frac{\sqrt{\mathrm{d}x^2 + \mathrm{d}y^2}}{y} = \int_0^1 \frac{\sqrt{\mathrm{d}(\cos(\theta(t)))^2 + \mathrm{d}(\sin(\theta(t)))^2}}{\sin(\theta(t))}$$
by direct computation. Give both an exact answer and an approximation sufficient to distinguish from the answer to question 2. **Do not be scared to ask for help in office hours on this one. It's trickier than it looks.**

   **Solution:** Insert solution here! Do the same for the other problems!

2. (2 points) Compare this to the hyperbolic length of $\gamma_0(t) = i\sqrt{3}/2 + (t - (1/2))$. Is $\gamma_0(t)$ a geodesic?

3. (3 points) Verify that after applying the matrix $\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ in $\mathrm{SO}_2(\mathbf{R})$, complex numbers of the form $\cos\theta + i\sin\theta$ (with $0 < \theta < \pi$) are moved to the imaginary axis. Use this to give the hyperbolic length of $\gamma$.

4. (5 points) Verify that if $a^2 + b^2 \neq 1$ and $\theta = \frac{-1}{2}\arctan(2a/(a^2+b^2-1))$ then $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ moves $a + bi \in \mathcal{H}$ to the imaginary axis.

5. (3 points) Compute the length of $\gamma$ one more time, this time with the cross-ratio. What method is easiest for you?

6. (5 points) Verify that the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbf{R})$ with $cd \neq 0$ sends the imaginary axis to the open half-circle centered on the real axis with limit points $a/c$ and $b/d$. Find the center and radius.