TOPICS IN MODERN GEOMETRY
HYPERBOLIC PROJECTS

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- **Lie Groups** Beyond just $\mathrm{SL}_2(\mathbf{R})$ and $\mathrm{SO}_2(\mathbf{R})$ there is a whole world of topological groups whose group laws are also differentiable. These are called Lie groups and they have a number of amazing properties. Within here, there are many options for you to pursue
  - **The Iwasawa Decomposition** The Iwasawa decomposition generalizes to many higher-dimensional groups. You might select one or try to say something more general.
  - **Discrete subgroups** What is it that's special about discrete subgroups of $\mathrm{SL}_2(\mathbf{R})$ or $\mathrm{PSL}_2(\mathbf{R})$? What is the analogue of Fuchsian groups in other domains?
- **Modular Forms** These are a huge area of study today with connections to just about every area of mathematics. There are very well-known connections to the theory of partitions, elliptic curves, quadratic forms, and there are even variants like Maass forms which don't need to be holomorphic. Make a proposal.
- **Fuchsian groups and fundamental domains** A longer course would involve using the fundamental domain of a Fuchsian group to find generators in a systematic way. How does one do this? Can we find generators for any special Fuchsian groups? What can we say about the Fuchsian groups whose fundamental domains have finite area?
- **Higher dimensional hyperbolic spaces** In class we showed how to realize the upper half plane as $\mathrm{SL}_2(\mathbf{R})/\mathrm{SO}_2(\mathbf{R})$. What is the right analogue for higher dimensions? What about if we require a complex structure?
- **Algebraic curves** What is the connection between algebraic curves over the complex numbers and the area of a fundamental domain of a Fuchsian group? This is a big topic, but there are many very good references.