TOPICS IN MODERN GEOMETRY
TOPOLOGY
HOMEWORK SHEET 2

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Exercise 1. Show that a metrisable topological space is Hausdorff.

Exercise 2. Let S^1 denote the circle, with the subspace topology from $\mathbb{C}$. Show that

- $S^1 \times [a, b]$ is homeomorphic to both the cylinder (given subspace topology from $\mathbb{R}^3$) and the annulus (given subspace topology from $\mathbb{R}^2$);
- $S^1 \times S^1$ with the product topology is homeomorphic to the torus given the quotient topology induced from the unit square $[0, 1]^2$, which in turn has the subspace topology from $\mathbb{R}^2$. Show that both of these topologies agree with the subspace topology from the usual embedding into $\mathbb{R}^3$.

Exercise 3. Let $(G, \cdot)$ and $(H, \times)$ be topological groups. Show that the direct product $G \times H$ is a topological group with respect to the product topology. Recall that the direct product is given by the set

$$G \times H = \{(g, h) : g \in G, h \in G\},$$

with group operation given by

$$(g_1, h_1)(g_2, h_2) = (g_1 \cdot g_2, h_1 \times h_2).$$

For the group theorists: Let N be a normal subgroup of a topological group G, show that the quotient group G/N is a topological group with respect to the quotient topology.

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