TOPICS IN MODERN GEOMETRY
TOPOLOGY PROBLEMS CLASS 2 EXERCISES

THOMAS OLIVER
UNIVERSITY OF BRISTOL

In this problems class we will work through exam-type questions based on the interactions of topology and algebra considered in the lectures 5-7.

Exercise 1. Show that $GL_2(\mathbb{R})$ is a locally compact topological space with respect to the topology induced from $\mathbb{R}^4$. Write down $\varepsilon > 0$ such that the following subset of $GL_2(\mathbb{R})$ forms a compact neighbourhood of $I \in \mathrm{GL}_2(\mathbb{R})$:

$$\{\begin{pmatrix} 1+a & b \\ c & 1+d \end{pmatrix} : a,b,c,d \in [-\varepsilon,\varepsilon]\}.$$

Exercise 2. Show that the closed interval $[a,b] \subseteq \mathbb{R}$ is connected with respect to the Eucludean topology. Now endow $\mathbb{R}$ with a topology whose basis is the collection of all intervals of the form $[a,b)$. Show that $\mathbb{R}$ is not connected with respect to this topology.

Exercise 3. Let X be a path-connected topological space. Show that X is connected.

Date: September - October, 2015.