

**TOPICS IN MODERN GEOMETRY
TOPOLOGY
HOMEWORK SHEET 1**

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Exercise 1. Let X be a set and say a subset U is open if $U = \emptyset$ or $X \setminus U$ is countable. Show that this assignment of open sets defines a topology on X . This is called the *co-countable topology*. Show that if X is countable then the co-countable topology agrees with the discrete topology on X .

Exercise 2. Explain how the fact that any polynomial function $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous can be deduced from continuity of the addition and multiplication maps $\mathbb{R}^2 \rightarrow \mathbb{R}$.

Exercise 3. Write down all open sets of $\{0, 1\}$ with respect to the discrete topology. How many open subsets does $\{0, 1, \dots, N\}$ have with respect to the discrete topology?