# “TOPICS IN MODERN GEOMETRY”
# TOPOLOGY PROJECTS

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**Completions of $\mathbb{Q}$.** Recall that the set of rational numbers $\mathbb{Q}$ is *dense* in $\mathbb{R}$. This is far from a unique situation. Indeed, there is an infinite family of fields in which $\mathbb{Q}$ is dense, namely, the fields $\mathbb{Q}_p$, in which $p$ is a prime number. One of the most important topological properties of $\mathbb{Q}_p$ is that it is a *locally compact abelian group*, which means in particular that one may develop an integration theory analogous to the Lebesgue measure on $\mathbb{R}$. In this project you will study some aspects of $\mathbb{Q}_p$ in more detail, for example, you could:

(1) Develop the rudiments of integration theory on $\mathbb{Q}_p$ and evaluate some special integrals;
(2) Study topological self-duality;
(3) Study fractal realisations of $\mathbb{Q}_p$ in Euclidean space.

**Zariski Topology.** Some tricksters will try to tell you that non-Hausdorff topological spaces are undesirable, even pathalogical. In this project you will study such a topology, namely the *Zariski topology*, and come to understand that it is really rather natural in the right context. The Zariski topology arises naturally on the solution sets of polynomial equations, and, despite its sometimes strange behaviour, this is the basic topology used in algebraic geometry. In this project you will study in detail some specific aspects of the Zariski topology, for example:

(1) Comparison to other topological structures on a given space, such as the subspace topology on a complex curve or the quotient topology on the projective plane;
(2) Why the zero loci of polynomials give rise naturally to the Zariski topology;
(3) Reformulations of standard concepts such as compactness to better suit the algebraic context.

**Projective Space.** In the lectures we saw the real projective plane as a quotient of the Mobius strip, and gave the definition for more general projective spaces. One may study geometry on the projective plane, and that is the intention of this project. For example, you could:

(1) Contrast properties of projective geometry to the more familiar Euclidean and hyperbolic settings;
(2) Study conic sections in the projective plane, with a view towards Pascal’s theorem or Poncelet’s porism;
(3) Develop the *finite geometry* of projective spaces over finite fields.

**Classification of Compact Surfaces.** In the lectures we stated that it was possible to classify compact two-dimensional manifolds up to homeomorphism. In this project you will work through the proof of this result.